

Localization of excited states of Bose-Einstein Condensates in presence of disorder

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Abstract. We study the onset of localization from excited states of trapped Bose-Einstein Condensates expanding in presence of Gaussian uncorrelated random disorder. In 1D systems, we observe that for a fixed ratio between the disorder strength and the initial energy, excited states localize exponentially with a localization length that decreases as the energy of the initial state increases. Moreover, the localized state keeps the shape of the initial state wave function with an exponential tail.

In 2D, we analyze the interplay between vorticity and localization by examining the dispersion of a state containing a vortex on it in a disordered media. Despite localization can be associated to islands of constant phase, the presence of a vortex in the initial state leads to dislocations and phase jumps in the localized state. The study of dispersion of a bosonic condensate with vorticity bears similarities to the stability of topological excitations in 2D p-wave fermionic superfluids.

Keywords: Bose-Einstein Condensation, Anderson localization, superfluids, vorticity

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1. Introduction

Disorder effects are ubiquitous in Nature and strongly determine subtle features such as insulating properties of materials, the striking behavior of high-T_c superconductors or the inhibition of transport and localization. Quantum localization can be formally defined, for unbounded systems, as the phenomenon in which static disorder on a single non-interactive particle Hamiltonian causes all its eigenstates to localize in space. The spectrum of the Hamiltonian is no longer continuous, but rather becomes dense point-like; the eigenvalues are infinitely close to each other and the eigenfunctions are exponentially localized and characterized by a localization length ξ , see e.g. [1] and references therein. Only in 1D has been rigorously demonstrated that no

matter how small the disorder is, it always leads to an exponential localization of all the eigenfunctions by repeated backscattering [2, 3]. Localization emerges as a consequence of the coherent back scattering that appears when a wave packet spreads in a disordered media [4]. In such cases, there are multiple interference paths which can add constructively. Those constructive paths, when averaged over the disorder, lead to an exponentially localized wave packet. Localization is, therefore, a universal wave phenomena which is associated to the suppression of transport together with an exponential wave packet localization. However, localization and transport inhibition can hardly be regarded as explicit features of Anderson quantum localization [4] since classical localization – occurring when a particle of energy E is surrounded by a disordered landscape with hills of higher energy– will also present such properties [5]. The quantum signatures of disorder are far more subtle. For instance, the classical localization length defined as the averaged size of the classically allowed paths, increases with the energy of the particle E for any model of disorder, while it has been shown that a proper tailoring of the disorder correlation can revert this feature for and only for quantum localization [5].

To date, most studies of quantum localization in ultracold gases have exhaustively analyzed the onset of localization for the ground state of a trapped gas expanding freely in a disordered media and also its effects in collective excitations [6,7]. Seminal ultracold gases experiments have demonstrated truly quantum localization effects by expanding a trapped BEC in a 1D waveguide with a very weak speckle potential superimposed on it –ensuring that interactions are highly inhibited– [8]. There, it was shown that localization corresponds to many quantum reflections of small amplitude, and so, in the presence of a disorder, each matter wave localizes leading to a density profile with exponentially decaying tails and a localization length corresponding to that of non-interacting particles with momentum $1/h_{heal}$, being h_{heal} the healing length. The localization lengths as a function of the strength of the disorder in this experiment are consistent with the existence of a cross-over from exponential to algebraic localization, and therefore with the theoretical prediction of [9] of the existence of an effective mobility edge at $k = 1/L_{dis}$, being k the atomic wavevector and L_{dis} the correlation length of the disorder. In [10], a very different route was taken to achieve quantum localization. First, particle interactions were suppressed by means of Feshbach resonances and then the condensate was left to expand in a quasi periodic lattice reproducing the Aubry-André model [11] or the non-interacting Harper model [12] of quantum localization.

Here, we ask ourselves for the effects of propagation of non trivial initial states in a disordered media. Explicitly we want to investigate the behavior of a superfluid supporting vortices or topological charges in such a medium. To this aim, we start by analyzing first the free expansion of the excited states of an ideal gas trapped in a harmonic potential in the presence of a Gaussian uncorrelated disorder. We observe that, no matter if the localization is classical or quantum, the localized state –after averaging over different disorder realizations– reproduces faithfully the density of the initial harmonic eigenstates together with exponential tails. This poses the

question if a superfluid with a vortex on it can localize, and if the vortex survives to the scattering process in the localization effect. Interestingly enough we numerically observe that for completely uncorrelated disorders, the phase of the superfluid after expanding in a disordered media is completely lost, but when the disorder is switched off and the superfluid is let to evolve freely, the phase which was present before the disorder reappears again and vorticity is preserved. Previous studies [13] in 1D and 2D BEC condensates have shown that the localization length, ξ , is linked to the Fourier components of the disorder potential at typical wave vector $k_e = \sqrt{2mE}/\hbar$, where m stands for the particle mass and E for its typical energy. Also, it has been shown that in 1D $1/\xi(E) \propto C(2k_E)/E$ where C refers to the disorder correlation function $C(\mathbf{x}) = \langle V_{dis}(\mathbf{x} + \mathbf{x}') \rangle - \langle V_{dis}(\mathbf{x}') \rangle^2$. For uncorrelated disorders, C is a constant that has no dependence on the momentum but, as we shall see, it has a clear dependence on the strength of the disorder. We observe that if the ratio between the disorder strength and the energy of the initial state, E_n , is kept constant and smaller than one, the localization length of the corresponding initial state ξ_n decreases as E_n increases, contrary to what happens for a constant uncorrelated disorder where both in the quantum and in the classical case, the localization length increases with increasing energy [5].

2. Method

The dynamics of the condensate at $T = 0$ is well described by a macroscopic wave function $\varphi(\mathbf{x}, t)$ that depends on position \mathbf{x} and time t and that fulfils the Gross-Pitaevskii equation (GPE, see for example [14, 15]):

$$i\hbar \frac{\partial \varphi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{trap} + V_{dis,i} + g|\varphi|^2 \right] \varphi \quad (1)$$

where m is the atomic mass and g is the effective self-interaction strength that depends on the three-dimensional scattering length and on the transverse confinement. In what follows we consider Rb^{87} and consider initially an ideal gas, i.e. $g = 0$. We assume the condensate to be trapped in a simple harmonic potential, $V_{trap} = \frac{1}{2}m\omega^2\mathbf{x}^2$ where ω is the trapping frequency, which in the 2D case, its components are taken to be equal $\omega = \omega_x = \omega_y$. The disorder potential V_{dis} is assumed to be spatially uncorrelated with a Gaussian distribution:

$$V_{dis,i}(\mathbf{x}, t) = \begin{cases} 0 & \text{if } t = 0 \\ Dn_i(\mathbf{x}) & \text{otherwise} \end{cases} \quad (2)$$

where $n_i(\mathbf{x})$ are a set of independent Gaussian random distributions (zero average and unit variance) for each realization of the disorder i and D is the strength of the disorder. Our numerical simulations of the GPE are based on finite-differences combined with the split-operator method. Our results for the density profiles are obtained after averaging over N different random realizations of the disorder potential $V_{dis,i}$.

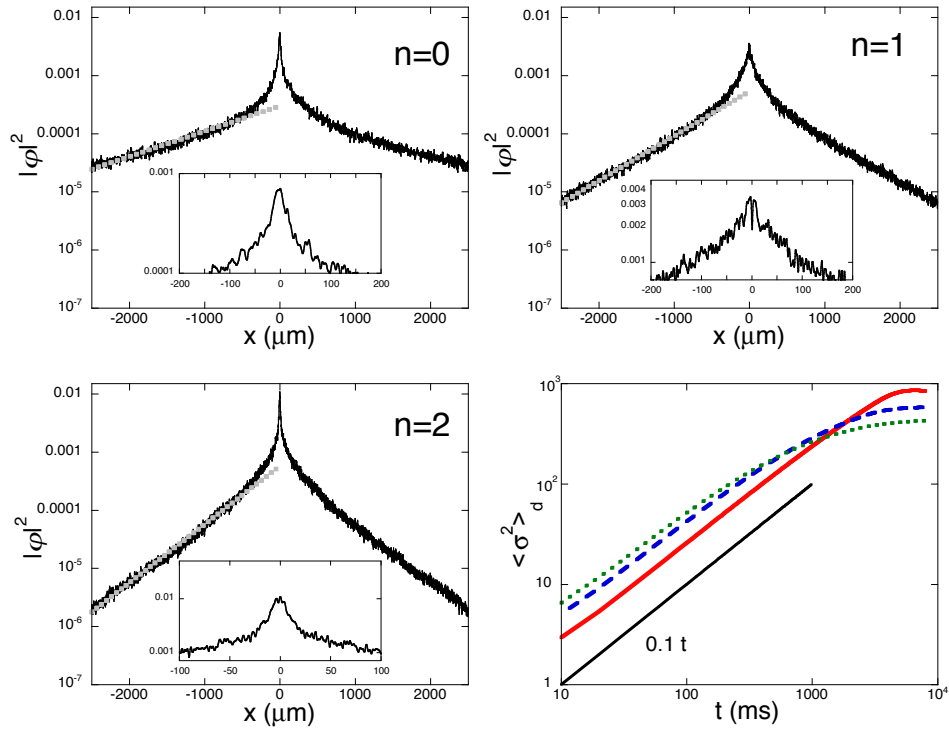


Figure 1. Density of the wavefunction $|\varphi_n(x)|^2$ (solid line) after diffusing for a time $t = 8\text{s}$. The system is initially prepared in the state $\varphi_n(x)$ with $n = 0$ (top-left), $n = 1$ (top-right) and $n = 2$ (bottom-left). The dashed lines are the best exponential fit functions $\exp[-x/L_n]$ (see text). On the insets the rescaled initial density distributions $|\varphi_n(x)|^2$ near $x = 0$ are shown. Bottom-right panel: variance $\langle \sigma^2 \rangle_d$ averaged over the disorder realizations as a function of time for the initial state $n = 0$ (thick solid), $n = 1$ (dashed), $n = 2$ (dotted). The numerical data are compared to a simple linear function $0.1t$ describing diffusion.

3. Results: 1D

In this section we show our results for the one-dimensional case in which, for simplicity, we assume non-interacting particles i.e. $g = 0$. At $t = 0$, the system is prepared in one of the eigenstates of the time-independent Hamiltonian (1) in absence of disorder, i.e. $D = 0$, i.e. whose eigenfunctions are

$$\begin{aligned} \varphi(x, 0) &= \varphi_n(x) \quad n = 0, 1, 2, \dots \\ \varphi_n(x) &\equiv \frac{1}{\sqrt{2^n n!}} \left(\frac{1}{\pi \lambda^2} \right)^{1/4} e^{-\frac{x^2}{2\lambda^2}} H_n \left(\frac{x}{\lambda} \right) \end{aligned} \quad (3)$$

where H_n are the Hermite polynomials and $\lambda = \sqrt{\hbar/m\omega}$ is the oscillator length. For the non-interacting case, we take the strength of the disorder D to be proportional to the energy of the initial eigenstate of the harmonic potential, i.e. $D/E_n = 0.52$. This permits a straight comparison between the localization length of the different initial states. To ensure that classical localization is avoided we further bound the disorder strength such that $V_{dis,i} = 0$ either if $Dn_i(\mathbf{x}) > 0.7E_n$ or $V_{dis,i} < 0$. The number of

disordered potentials used for the averages in 1D is $N = 100$. At $t = 0$, the trapping potential V_{trap} is suddenly switched off while the disordered potential is turned on. In the absence of any disorder, if the initial wave packet is Gaussian, the wave packet shows ballistic expansion, with the center of mass remaining at zero velocity and the width increasing linearly with time. To signal the onset of localization we allow the wave packet to evolve in time until the dispersion of the initial wave packet averaged over the disorder realizations remains constant, i.e. $\langle \sigma^2 \rangle_d = \langle \langle x^2 \rangle - \langle x \rangle^2 \rangle_d$ does not depend on time. When this situation is reached, we monitor the wave packet density distribution and observe that its shape remains constant for longer times. The wave-function diffuses for a time approximately of $t = 8s$ under the influence of just the disorder potential $V_{dis,i}$ and the final density is averaged over $N = 100$ different realizations of the disorder. As shown in Fig. 1, the final density $|\varphi(x)|^2$ averaged over the disorder shows exponential tails, while the central part of the localized wavefunction, as shown in the corresponding insets, reproduces the nodes of the initial eigenstate of the harmonic potential. This is witnessed by the number of peaks around $x = 0$ as compared to the initial wave function. An exponential ansatz of the form $|\varphi(x)|^2 = A_n \exp[-x/\xi_n]$ gives the best-fit results $\xi_0 = 1300\mu m$; $\xi_1 = 580\mu m$; $\xi_2 = 440\mu m$, indicating that the localization length, ξ_n decreases with increasing energy. We have also analyzed the evolution of the variance of the density distribution as a function of time for the three initial states with $n = 0, 1, 2$. The numerical simulations show a ballistic expansion for very short times (not shown in the figures) followed by a clear diffusive behavior $\langle \sigma^2 \rangle_d \approx t$ finally leading to a saturation for longer times. If we repeat the simulations for a much higher value of the disorder potential, so that $D \gg E_n$, several features remain, as for instance, the shape of the wave function near the origin and the exponential tails of the wavefunction, localization occurs at much shorter time scale but the localization length ξ_n becomes independent of the initial eigenstate, evidencing that classical localization takes place. In such cases, ansatzs of the form $|\varphi_n(x)|^2 = B_n x^{3/2} \exp[-x/S]$ give a good fit of the the overall wavefunction [16], where S plays now the role of a localization length and its value is, to all effects, independent on the initial state n . Finally, for a sufficiently large particle interactions parametrized by a value $g \neq 0$, the onset of localization and the memory effects disappear, as expected.

4. Results: 2D

As it is well recognized [17,18], the dynamical behavior of even non-interacting particles in disorder potentials is a difficult problem in dimensions higher than one. A detailed analysis of classical localization, with diffusive, super diffusive and even subdiffusive behavior has been recently presented for random correlated disorder in the regime where $\lambda_{dB} \ll \sigma_r \leq l_B \ll L$ where λ_{dB} corresponds to the atomic de Broglie wavelength, σ_r is the correlation length of the disorder, l_B is the mean free path (Boltzmann) and L the size of the system. Under the above circumstances, it has been shown [18], that a variety of diffusion regimes appear depending on the disorder properties. In the 2D case

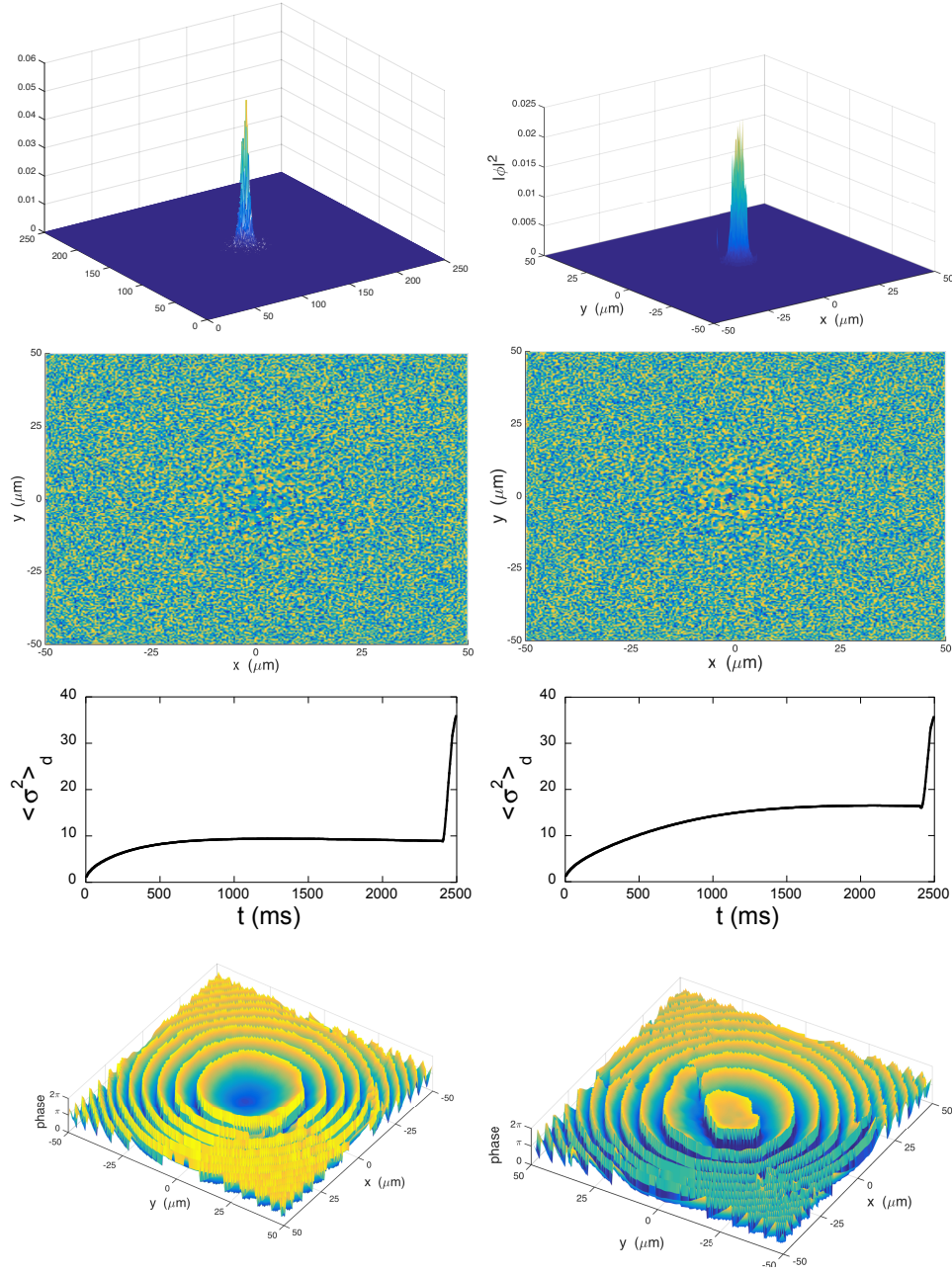


Figure 2. The left column corresponds to the initial state $\varphi_{0,0}(x, y)$. From top to bottom (i) wave function density after expanding freely in the presence only of the disordered potential at $t = 2.45s$, (ii) phase of the wave function at $t = 2.45s$, (iii) variance $\langle \sigma^2 \rangle_d$ of the position as a function of time, (iv) phase of the wave function at $t = 2.5s$ after switching off the disorder and let the condensate evolve freely for 5ms. The left column corresponds to the initial state $\varphi_{0,0}$, and right column corresponds to the initial state $\varphi_{1,1}$. All the figures are averaged quantities over disorder realizations.

we solve the equation:

$$i\hbar \frac{\partial \varphi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{trap} + V_{dis,i} + g|\varphi|^2\varphi - L_z\Omega \right] \quad (4)$$

We consider as initial states the stationary states of a rotating condensate in absence of disorder and interactions; $\varphi_{0,0}(x,y) = \exp(-\rho^2/2\lambda^2)/\sqrt{\pi\lambda^2}$ with $\rho^2 = x^2 + y^2$ corresponding to $(n = 0, l = 0)$ or the first excited state $\varphi_{1,1}(x,y) = (x + iy)\exp(-\rho^2/2\lambda^2)/\sqrt{\pi\lambda^2}$ ($n = 1, l = 1$) with a vortex in it. The procedure we follow is as before; first we calculate the eigenstates of the 2D trap (both for a non-interacting and for an interacting gas) in the absence of disorder, then we release the condensate initially prepared in one of the eigenstates of the Hamiltonian by switching off trapping potentials and switching on instantaneously the disordered potential $V_{dis,i}$ modeled again by an uncorrelated gaussian distribution of strength D . In the 2D case, there is not need to limit the strength of the disorder to lower values than the energy of the eigenstate since classical localization is not guaranteed for uncorrelated disordered in such geometry. Therefore, we fix $D/E_{n,l} = 6$ since simulations in 2D for smaller disorder strength are accompanied by much longer times to achieve localization which in turn demands prohibitively large grid sizes. We use $N = 60$ different disorder realizations for each initial state and calculate the variance of the position. When the variance remains constant over time, so that the diffusive character of the initial expansion has ended, we keep the density of the BEC and its phase as depicted in the upper panels of Fig 2 and average our results over all disorder realizations. The wave packet after $t = 2s$ has clearly localized for both, the ground and the excited state, while the phase of the BEC at t displays an absolutely random pattern in both cases as shown in the second row of Fig. 2. However, if we switching off the disorder and let the BEC to expand again freely, immediately the localized wave packet expands (as shown by the variances, see Fig. 2) but the phase of the condensate, when averaged over the disorder, immediately recovers a well defined structure. Our results are shown in the lower panels of Fig.2. The ground state ($n = 0, l = 0$), localizes very rapidly and the average phase of the condensate becomes completely random. After the disorder has been switched off, the phase of the condensate initially in the ground state, reproduces the propagation of an initial gaussian state that is let to evolve freely and we recover the self-similar solution, with the phase expanding from zero to 2π centered at the center of the original harmonic trap and concentric rings repeating the pattern with the phase growing from 0 to 2π . The wavepacket after the disorder has been switch off is not anymore a gaussian state. For the excited state ($n = 1, l = 1$) a similar effect occurs. The initial state with a vortex on it stabilizes and presents a completely random phase in the presence of disorder (see right column Fig. 2). The average phase over all disorder distributions is also random. As soon as the disorder is switched off and the wave function is left to evolve freely for 5ms, the phase arranges and presents a plateau of constant phase followed by a clear discontinuity on the phase near the origen which corresponds to the original vortex. The helicoidal structure in the phase is due to the rotation of the condensate. In the presence of interactions, the above pictures brake down completely and one cannot recover the phase pattern that reproduces neither the expansion of a Gaussian state ($n = 0, l = 0$) or of the vortex ($n = 1, l = 1$) and the average phase is not well defined at any time.

5. Summary

We have presented here a numerical study on the effects of random uncorrelated disorder in non trivial initial states. We have analyzed the onset of localization, both classical and quantum for a 1D gas with and without interactions, starting from an excited state. We have observed that for both cases, and for such disorder model, the averaged wave function keeps memory of the initial state in which it was let to evolve and show the typical exponential tails. If the disorder is forced to be always smaller than the energy of the initial state to avoid classical confining, the exponential length of the localized state decreases with the energy of the initial state. For a classical localization this is not the case. Our study has mainly address the onset of localization for a superfluid containing a vortex on it. Even for non-interactive particles, numerical restrictions force to relax the disorder strength in 2D. In the 2D case, we observe that for a relative small disorder –but with a landscape of disorder that can include also hills of larger energy than the energy of the stationary initial state–, the wave function localizes in the disordered media as shown by the average dispersion as a function of time and the averaged phase of the wave packet becomes completely random. Nevertheless, as soon as the disorder is switched off and free propagation is allowed the wave packet phase recovers the initial features and shows a self similar solution with concentrating rings with the phase going from 0 to 2π for the initial Gaussian state, or the vortex with a helicoidal structure signaling the rotation present at $t = 0$.

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